CONNECTIVITY IN ONE-DIMENSIONAL AD HOC NETWORKS WITH AN ACCESS POINT

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Abstract. In this paper, we study the connectivity in one-dimensional ad hoc wireless networks with an fixed access point. In recent years, various closed expressions for the probability of connectivity on one-dimensional networks (interval graphs) have been derived by many researchers. We will provide some numerical validation for them by means of extensive simulations.

Keywords: interval graph; ad hoc network; connectivity; component.

1. Introduction

Recently, ad hoc networks have attracted extensive research interest within computer communication and engineering communities. Connectivity of the underlying network is the foundation for its functions, as is indicated in [2, 9]. A number of mathematically rigorous results on the asymptotic critical transmission radius and asymptotic critical neighbors for the connectivity of network in one-dimensional areas have been obtained, see e.g. [1, 3, 4, 5, 7, 8, 11, 12]. The random version of one-dimensional network is also called the random interval graph, which has been studied in depth in [6]. More recently, a concept of access points are introduced in [10]. Given $L, r > 0$, let $X_1, \cdots, X_n$ be n independent uniformly distributed random variables on the interval $[0, L]$. Denote by $G(n, L)$ the graph with vertex set $\{X_1, \cdots, X_n\}$ and with an edge X_iX_j $(i \neq j)$, if $|X_i - X_j| \leq r$. Some access points $\{z_j\}$ can exist in $G(n, L)$. An interesting concept of accessible connectivity has also been introduced, which is different from the ordinary connectivity with possible fixed nodes. Multiple components issues are considered in [8] ($cf.$ Theorem 2.2 below).

In this paper we will perform numerical study on the connectivity of one-dimensional ad hoc networks with a fixed point. Our work can be viewed as an independent test of the theoretical results obtained in prior work.

2. Numerical study

We need some further definitions here. We denote the graph by $G_x(n, L)$ if there exists a fixed node at the point $x \in [0, L]$. In particular, when $x = 0$, the graph is $G_0(n, L)$. We use $Q_m(\cdot)$ to denote the probability that the above graph model is composed of exactly m components. The following results are known.

Theorem 2.1. *([4, 7]) We have*

$$Q_1(G(n,L)) = \sum_{i=0}^{k_1} (-1)^i \binom{n-1}{i} (1 - \frac{ir}{L})^n, \tag{2.1}$$

and

$$Q_1(G_0(n,L)) = \sum_{i=0}^{k_2} (-1)^i \binom{n}{i} (1 - \frac{ir}{L})^n, \tag{2.2}$$

where $k_1 = n - 1 \wedge \lfloor L/r \rfloor$ *and* $k_2 = n \wedge \lfloor L/r \rfloor$*, respectively.*

More generally, the following statements are true.

Theorem 2.2. *([8]) Let m be a natural number. We have*

$$Q_m(G(n,L)) = \sum_{i=m-1}^{k_1} (-1)^{i-m+1} \binom{i}{m-1} \binom{n-1}{i} (1 - \frac{ir}{L})^n, \tag{2.3}$$

and

$$Q_m(G_0(n,L)) = \sum_{i=m-1}^{k_2} (-1)^{i-m+1} \binom{i}{m-1} \binom{n}{i} (1 - \frac{ir}{L})^n, \tag{2.4}$$

where $k_1 = n - 1 \wedge \lfloor L/r \rfloor$ *and* $k_2 = n \wedge \lfloor L/r \rfloor$*, respectively.*

To test the results in Theorem 2.2, we have performed extensive simulations on graph $G(n, L)$ with a fixed node at 0. For different values of n and m, we plot the probability Q_m as a function of L/r, see Fig.1 –Fig. 6. A future research problem would be to derive exact formula on two dimensional ad hoc networks. This issue is very challenging.

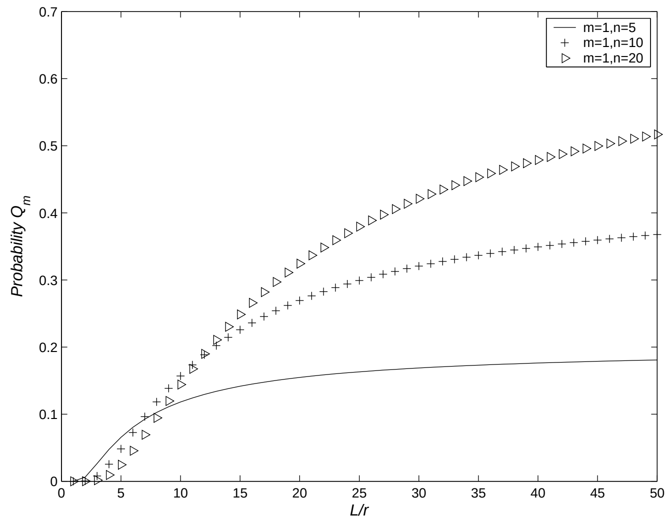

FIGURE 1. The probability $Q_m(G_0(n,L))$ as a function of L/r for $m = 1$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

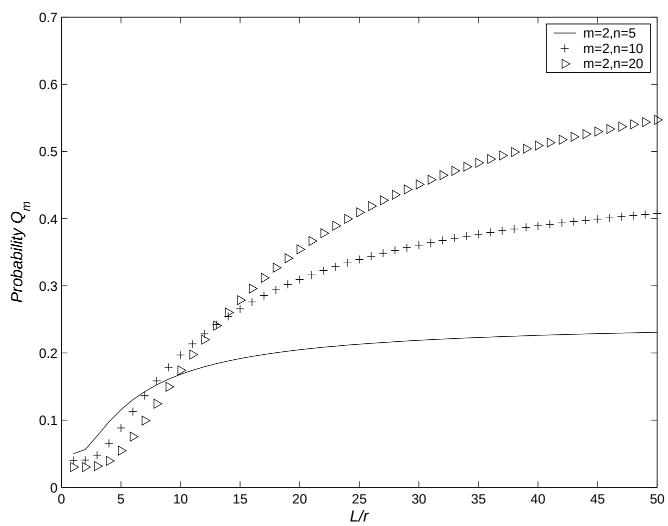

FIGURE 2. The probability $Q_m(G_0(n,L))$ as a function of L/r for $m = 2$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

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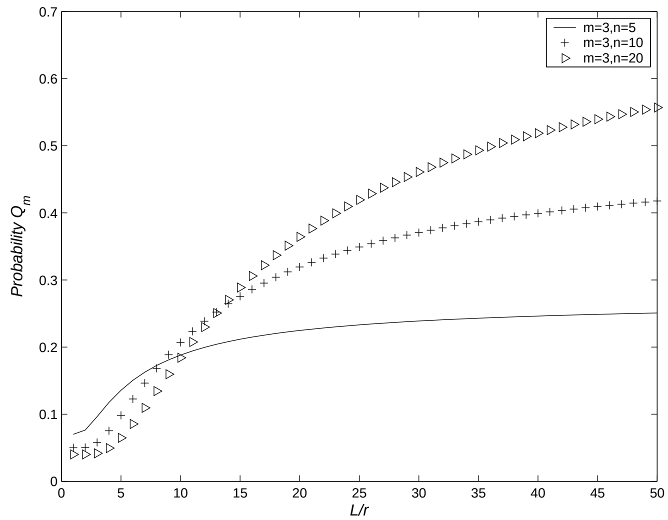

FIGURE 3. The probability $Q_m(G_0(n, L))$ as a function of L/r for $m = 3$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

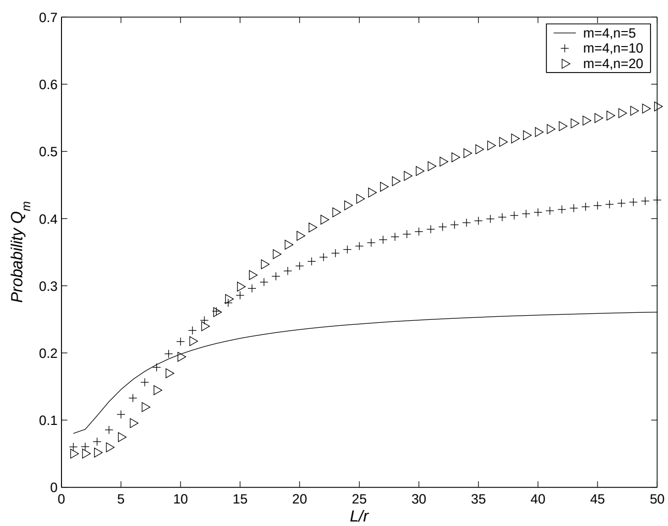

FIGURE 4. The probability $Q_m(G_0(n, L))$ as a function of L/r for $m = 4$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

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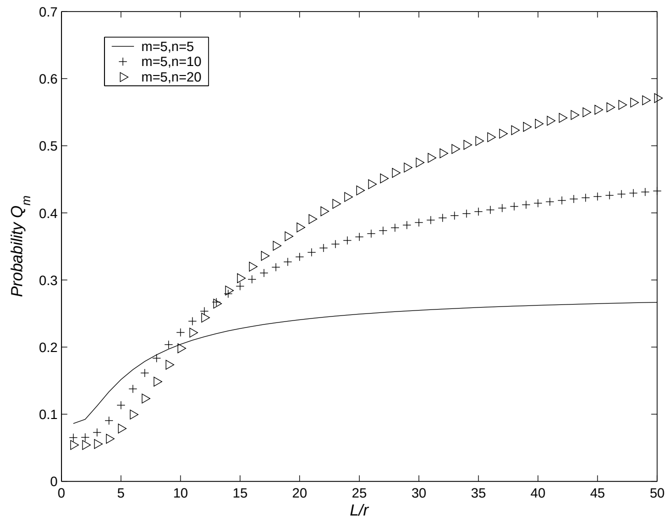

FIGURE 5. The probability $Q_m(G_0(n, L))$ as a function of L/r for $m = 5$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

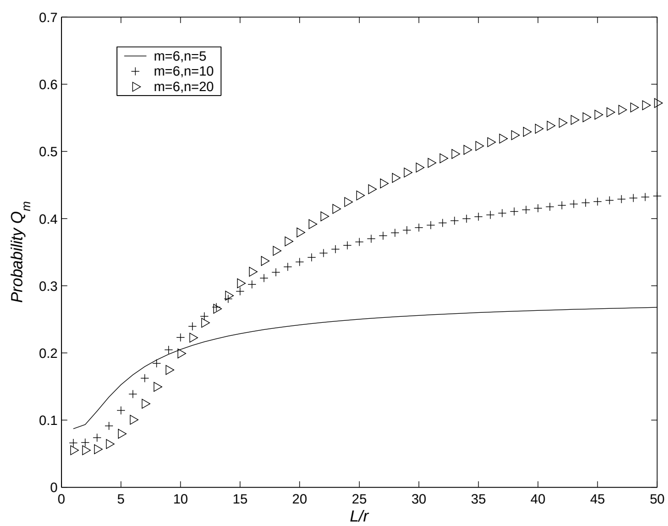

FIGURE 6. The probability $Q_m(G_0(n, L))$ as a function of L/r for $m = 6$ and different values of n: $n = 5$ (solid curves), $n = 10$ (pluses) and $n = 20$ (triangles).

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